DRŽAVNO NATJECANJE IZ LOGIKE 2024.
A KATEGORIJA
RJEŠENJA

Zadatak 1.

DA, NE

Ukupno 6 bodova.

Zadatak 2.

	Formula	(a)	(b)	(c)	(d)
i.	$\exists x \forall y (\neg Pyx)$	I	N	N	I
ii.	$\forall x \exists y \forall z ((Szx \rightarrow \neg Szy) \wedge (\neg Szy \rightarrow Szx))$	N	I	N	N
iii.	$\exists x \exists y \exists z \exists u (\neg ((Sxy \wedge Szu) \rightarrow \neg Syz))$	I	I	I	I
iv.	$\forall x \forall y \forall z ((Bx \wedge By \wedge \forall u ((Pux \vee Puy) \rightarrow Puz)) \rightarrow Bz)$	N	N	N	N
v.	$\forall x ((\forall y (Pyx)) \rightarrow Bx)$	I	N	I	I
vi.	$\forall x \exists y (By \wedge Pyx \wedge \forall z ((Bz \wedge Pzx) \rightarrow Pzy))$	N	I	N	N
vii.	$\forall x \forall y \forall z (Pxz \vee \neg (Pxy \vee Pyx))$	N	I	N	N
viii.	$\forall x \forall y (\neg Pxy \rightarrow (\neg Pyx \rightarrow (\neg Pxy \rightarrow Pyx)))$	N	I	N	N
ix.	$\forall x (\neg \exists u Sxu \vee \exists y \exists z (Sxy \wedge Sxz))$	I	I	I	I

Ukupno 72 boda.

Zadatak 3.

Dedukcija: prva varijanta

1	$\forall\,\textcircled{o}\,\forall x\forall y\exists z\; x\,\textcircled{o}\,y=z$	pretp.
2	$\forall x\forall y\; x\,\textcircled{ℓ}\,y=x$	pretp.
3	$\forall x\forall y(x\,\textcircled{$\neg$}\,y=\textcircled{$\top$}\leftrightarrow\neg x\,\textcircled{ℓ}\,y=\textcircled{$\top$})$	pretp.
4	$\forall x\forall y(x\,\textcircled{$\vee$}\,y=\textcircled{$\top$}\leftrightarrow(x\,\textcircled{ℓ}\,y=\textcircled{$\top$}\vee y\,\textcircled{ℓ}\,x=\textcircled{$\top$}))$	pretp.
5	$\forall\,\textcircled{$o_1$}\,\forall\,\textcircled{o_2}\,\forall\,\textcircled{o_3}\,\exists\,\textcircled{o_4}\,\forall x\forall y(x\,\textcircled{o_4}\,y=\textcircled{$\top$}\leftrightarrow\exists u\exists v((x\,\textcircled{o_1}\,y=u\wedge x\,\textcircled{o_2}\,y=v)\wedge u\,\textcircled{o_3}\,v=\textcircled{$\top$}))$	pretp.
6	$\forall\,\textcircled{$o_2$}\,\forall\,\textcircled{o_3}\,\exists\,\textcircled{o_4}\,\forall x\forall y(x\,\textcircled{o_4}\,y=\textcircled{$\top$}\leftrightarrow\exists u\exists v((x\,\textcircled{ℓ}\,y=u\wedge x\,\textcircled{o_2}\,y=v)\wedge u\,\textcircled{o_3}\,v=\textcircled{$\top$}))$	$\forall$i, 5
7	$\forall\,\textcircled{$o_3$}\,\exists\,\textcircled{o_4}\,\forall x\forall y(x\,\textcircled{o_4}\,y=\textcircled{$\top$}\leftrightarrow\exists u\exists v((x\,\textcircled{ℓ}\,y=u\wedge x\,\textcircled{$\neg$}\,y=v)\wedge u\,\textcircled{o_3}\,v=\textcircled{$\top$}))$	$\forall$i, 6
8	$\exists\,\textcircled{$o_4$}\,\forall x\forall y(x\,\textcircled{o_4}\,y=\textcircled{$\top$}\leftrightarrow\exists u\exists v((x\,\textcircled{ℓ}\,y=u\wedge x\,\textcircled{$\neg$}\,y=v)\wedge u\,\textcircled{$\vee$}\,v=\textcircled{$\top$}))$	$\forall$i, 7
9	$\forall x\forall y(x\,\textcircled{$+$}\,y=\textcircled{$\top$}\leftrightarrow\exists u\exists v((x\,\textcircled{ℓ}\,y=u\wedge x\,\textcircled{$\neg$}\,y=v)\wedge u\,\textcircled{$\vee$}\,v=\textcircled{$\top$}))$	pretp.
10	$\forall y(c\,\textcircled{$+$}\,y=\textcircled{$\top$}\leftrightarrow\exists u\exists v((c\,\textcircled{ℓ}\,y=u\wedge c\,\textcircled{$\neg$}\,y=v)\wedge u\,\textcircled{$\vee$}\,v=\textcircled{$\top$}))$	$\forall$i, 9
11	$c\,\textcircled{$+$}\,d=\textcircled{$\top$}\leftrightarrow\exists u\exists v((c\,\textcircled{ℓ}\,d=u\wedge c\,\textcircled{$\neg$}\,d=v)\wedge u\,\textcircled{$\vee$}\,v=\textcircled{$\top$}))$	$\forall$i, 10
12	$\neg(c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}\vee\neg c\,\textcircled{ℓ}\,d=\textcircled{$\top$})$	pretp.
13	$c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}$	pretp.
14	$c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}\vee\neg c\,\textcircled{ℓ}\,d=\textcircled{$\top$}$	$\vee$u, 13
15	$\bot$	$\bot$u, 14, 12
16	$\neg c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}$	$\neg$u, 13 – 15
17	$c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}\vee\neg c\,\textcircled{ℓ}\,d=\textcircled{$\top$}$	$\vee$u, 16
18	$\bot$	$\bot$u, 17, 12
19	$\neg\neg(c\,\textcircled{ℓ}\,d=\textcircled{$\top$}\vee\neg c\,\textcircled{ℓ}\,d=\textcircled{$\top$})$	$\neg$u, 12 – 18
20	$c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}\vee\neg c\,\textcircled{ℓ}\,d=\textcircled{$\top$}$	$\neg$i, 19
21	$c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}$	pretp.
22	$\forall x\forall y\exists z\; x\,\textcircled{$\neg$}\,y=z$	$\forall$i, 1
23	$\forall y\exists z\; c\,\textcircled{$\neg$}\,y=z$	$\forall$i, 22
24	$\exists z\; c\,\textcircled{$\neg$}\,d=z$	$\forall$i, 23
25	$c\,\textcircled{$\neg$}\,d=f$	pretp.
26	$c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}\wedge c\,\textcircled{$\neg$}\,d=f$	$\wedge$u, 21, 25
27	$\forall y(\textcircled{$\top$}\,\textcircled{$\vee$}\,y=\textcircled{$\top$}\leftrightarrow(\textcircled{$\top$}\,\textcircled{ℓ}\,y=\textcircled{$\top$}\vee y\,\textcircled{ℓ}\,\textcircled{$\top$}=\textcircled{$\top$}))$	$\forall$i, 4
28	$\textcircled{$\top$}\,\textcircled{$\vee$}\,f=\textcircled{$\top$}\leftrightarrow(\textcircled{$\top$}\,\textcircled{ℓ}\,f=\textcircled{$\top$}\vee f\,\textcircled{ℓ}\,\textcircled{$\top$}=\textcircled{$\top$})$	$\forall$i, 27
29	$\forall y\;\textcircled{$\top$}\,\textcircled{$\ell$}\,y=\textcircled{$\top$}$	$\forall$i, 2
30	$\textcircled{$\top$}\,\textcircled{$\ell$}\,f=\textcircled{$\top$}$	$\forall$i, 29
31	$\textcircled{$\top$}\,\textcircled{$\ell$}\,f=\textcircled{$\top$}\vee f\,\textcircled{$\ell$}\,\textcircled{$\top$}=\textcircled{$\top$}$	$\vee$u, 30
32	$\textcircled{$\top$}\,\textcircled{$\vee$}\,f=\textcircled{$\top$}$	$\leftrightarrow$i, 28, 31
33	$(c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}\wedge c\,\textcircled{$\neg$}\,d=f)\wedge\textcircled{$\top$}\,\textcircled{$\vee$}\,f=\textcircled{$\top$}$	$\wedge$u, 26, 32
34	$\exists v((c\,\textcircled{ℓ}\,d=\textcircled{$\top$}\wedge c\,\textcircled{$\neg$}\,d=v)\wedge\textcircled{$\top$}\,\textcircled{$\vee$}\,v=\textcircled{$\top$})$	$\exists$u, 33
35	$\exists u\exists v((c\,\textcircled{ℓ}\,d=u\wedge c\,\textcircled{$\neg$}\,d=v)\wedge u\,\textcircled{$\vee$}\,v=\textcircled{$\top$})$	$\exists$u, 34
36	$\exists u\exists v((c\,\textcircled{ℓ}\,d=u\wedge c\,\textcircled{$\neg$}\,d=v)\wedge u\,\textcircled{$\vee$}\,v=\textcircled{$\top$})$	$\exists$i, 24, 25 – 35
37	$\neg c\,\textcircled{$\ell$}\,d=\textcircled{$\top$}$	pretp.
38	$\forall x\forall y\exists z\; x\,\textcircled{ℓ}\,y=z$	$\forall$i, 1
39	$\forall y\exists z\; c\,\textcircled{ℓ}\,y=z$	$\forall$i, 38
40	$\exists z\; c\,\textcircled{ℓ}\,d=z$	$\forall$i, 39
41	$c\,\textcircled{$\ell$}\,d=g$	pretp.
42	$\forall y(c\,\textcircled{$\neg$}\,y=\textcircled{$\top$}\leftrightarrow\neg c\,\textcircled{ℓ}\,y=\textcircled{$\top$})$	$\forall$i, 3
43	$c\,\textcircled{$\neg$}\,d=\textcircled{$\top$}\leftrightarrow\neg c\,\textcircled{ℓ}\,d=\textcircled{$\top$}$	$\forall$i, 42
44	$c\,\textcircled{$\neg$}\,d=\textcircled{$\top$}$	$\leftrightarrow$i, 43, 37
45	$c\,\textcircled{$\ell$}\,d=g\wedge c\,\textcircled{$\neg$}\,d=\textcircled{$\top$}$	$\wedge$u, 41, 44
46	$\forall y(g\,\textcircled{$\vee$}\,y=\textcircled{$\top$}\leftrightarrow(g\,\textcircled{ℓ}\,y=\textcircled{$\top$}\vee y\,\textcircled{ℓ}\,g=\textcircled{$\top$}))$	$\forall$i, 4
47	$g\,\textcircled{$\vee$}\,\textcircled{$\top$}=\textcircled{$\top$}\leftrightarrow(g\,\textcircled{ℓ}\,\textcircled{$\top$}=\textcircled{$\top$}\vee\textcircled{$\top$}\,\textcircled{ℓ}\,g=\textcircled{$\top$})$	$\forall$i, 46
48	$\forall y\;\textcircled{$\top$}\,\textcircled{$\ell$}\,y=\textcircled{$\top$}$	$\forall$i, 2
49	$\textcircled{$\top$}\,\textcircled{$\ell$}\,g=\textcircled{$\top$}$	$\forall$i, 48
50	$g\,\textcircled{$\ell$}\,\textcircled{$\top$}=\textcircled{$\top$}\vee\textcircled{$\top$}\,\textcircled{$\ell$}\,g=\textcircled{$\top$}$	$\vee$u, 49
51	$g\,\textcircled{$\vee$}\,\textcircled{$\top$}=\textcircled{$\top$}$	$\leftrightarrow$i, 47, 50
52	$(c\,\textcircled{$\ell$}\,d=g\wedge c\,\textcircled{$\neg$}\,d=\textcircled{$\top$})\wedge g\,\textcircled{$\vee$}\,\textcircled{$\top$}=\textcircled{$\top$}$	$\wedge$u, 45, 51
53	$\exists v((c\,\textcircled{ℓ}\,d=g\wedge c\,\textcircled{$\neg$}\,d=v)\wedge g\,\textcircled{$\vee$}\,v=\textcircled{$\top$})$	$\exists$u, 52
54	$\exists u\exists v((c\,\textcircled{ℓ}\,d=u\wedge c\,\textcircled{$\neg$}\,d=v)\wedge u\,\textcircled{$\vee$}\,v=\textcircled{$\top$})$	$\exists$u, 53
55	$\exists u\exists v((c\,\textcircled{ℓ}\,d=u\wedge c\,\textcircled{$\neg$}\,d=v)\wedge u\,\textcircled{$\vee$}\,v=\textcircled{$\top$})$	$\exists$i, 40, 41 – 54
56	$\exists u\exists v((c\,\textcircled{ℓ}\,d=u\wedge c\,\textcircled{$\neg$}\,d=v)\wedge u\,\textcircled{$\vee$}\,v=\textcircled{$\top$})$	$\vee$i, 20, 21 – 36, 37 – 55
57	$c\,\textcircled{$+$}\,d=\textcircled{$\top$}$	$\leftrightarrow$i, 11, 56
58	$\forall y\; c\,\textcircled{$+$}\,y=\textcircled{$\top$}$	$\forall$u, 57
59	$\forall x\forall y\; x\,\textcircled{$+$}\,y=\textcircled{$\top$}$	$\forall$u, 58
60	$\exists\,\textcircled{o}\,\forall x\forall y\; x\,\textcircled{o}\,y=\textcircled{$\top$}$	$\exists$u, 59
61	$\exists\,\textcircled{o}\,\forall x\forall y\; x\,\textcircled{o}\,y=\textcircled{$\top$}$	$\exists$i, 8, 9 – 60

Druga varijanta

1	$\forall \textcircled{o}\,\forall x\forall y\exists z\; x \textcircled{o} y = z$	pretp.
2	$\forall x\forall y\; x \textcircled{ℓ} y = x$	pretp.
3	$\forall x\forall y(x \textcircled{$\neg$} y = \textcircled{$\top$} \leftrightarrow \neg x \textcircled{ℓ} y = \textcircled{$\top$})$	pretp.
4	$\forall x\forall y(x \textcircled{$\vee$} y = \textcircled{$\top$} \leftrightarrow (x \textcircled{ℓ} y = \textcircled{$\top$} \vee y \textcircled{ℓ} x = \textcircled{$\top$}))$	pretp.
5	$\forall \textcircled{$o_1$}\forall \textcircled{$o_2$}\forall \textcircled{$o_3$}\exists \textcircled{$o_4$}\forall x\forall y(x \textcircled{o_4} y = \textcircled{$\top$} \leftrightarrow \exists u\exists v((x \textcircled{o_1} y = u \wedge x \textcircled{o_2} y = v) \wedge u \textcircled{o_3} v = \textcircled{$\top$}))$	pretp.
6	$\exists y((a \textcircled{$\neg$} e = \textcircled{$\top$} \leftrightarrow y \textcircled{ℓ} e = \textcircled{$\top$}) \wedge (c \textcircled{ℓ} e = \textcircled{$\top$} \leftrightarrow y \textcircled{$\neg$} e = \textcircled{$\top$}))$	pretp.
7	$(a \textcircled{$\neg$} e = \textcircled{$\top$} \leftrightarrow b \textcircled{ℓ} e = \textcircled{$\top$}) \wedge (c \textcircled{ℓ} e = \textcircled{$\top$} \leftrightarrow b \textcircled{$\neg$} e = \textcircled{$\top$})$	pretp.
8	$a \textcircled{$\neg$} e = \textcircled{$\top$} \leftrightarrow b \textcircled{ℓ} e = \textcircled{$\top$}$	$\wedge$i, 7
9	$c \textcircled{$\ell$} e = \textcircled{$\top$} \leftrightarrow b \textcircled{$\neg$} e = \textcircled{$\top$}$	$\wedge$i, 7
10	$\forall y(a \textcircled{$\neg$} y = \textcircled{$\top$} \leftrightarrow \neg a \textcircled{ℓ} y = \textcircled{$\top$})$	$\forall$i, 3
11	$a \textcircled{$\neg$} e = \textcircled{$\top$} \leftrightarrow \neg a \textcircled{ℓ} e = \textcircled{$\top$}$	$\forall$i, 10
12	$\forall y(b \textcircled{$\neg$} y = \textcircled{$\top$} \leftrightarrow \neg b \textcircled{ℓ} y = \textcircled{$\top$})$	$\forall$i, 3
13	$b \textcircled{$\neg$} e = \textcircled{$\top$} \leftrightarrow \neg b \textcircled{ℓ} e = \textcircled{$\top$}$	$\forall$i, 12
14	$a \textcircled{$\ell$} e = \textcircled{$\top$}$	pretp.
15	$b \textcircled{$\ell$} e = \textcircled{$\top$}$	pretp.
16	$a \textcircled{$\neg$} e = \textcircled{$\top$}$	$\leftrightarrow$i, 8, 15
17	$\neg a \textcircled{$\ell$} e = \textcircled{$\top$}$	$\leftrightarrow$i, 11, 16
18	$\bot$	$\bot$u, 14, 17
19	$\neg b \textcircled{$\ell$} e = \textcircled{$\top$}$	$\neg$u, 15 – 18
20	$b \textcircled{$\neg$} e = \textcircled{$\top$}$	$\leftrightarrow$i, 13, 19
21	$c \textcircled{$\ell$} e = \textcircled{$\top$}$	$\leftrightarrow$i, 9, 20
22	$c \textcircled{$\ell$} e = \textcircled{$\top$}$	pretp.
23	$\neg a \textcircled{$\ell$} e = \textcircled{$\top$}$	pretp.
24	$a \textcircled{$\neg$} e = \textcircled{$\top$}$	$\leftrightarrow$i, 11, 23
25	$b \textcircled{$\neg$} e = \textcircled{$\top$}$	$\leftrightarrow$i, 9, 22
26	$b \textcircled{$\ell$} e = \textcircled{$\top$}$	$\leftrightarrow$i, 8, 24
27	$\neg b \textcircled{$\ell$} e = \textcircled{$\top$}$	$\leftrightarrow$i, 13, 25
28	$\bot$	$\bot$u, 26, 27
29	$\neg\neg a \textcircled{$\ell$} e = \textcircled{$\top$}$	$\neg$u, 23 – 28
30	$a \textcircled{$\ell$} e = \textcircled{$\top$}$	$\neg$i, 29
31	$a \textcircled{$\ell$} e = \textcircled{$\top$} \leftrightarrow c \textcircled{ℓ} e = \textcircled{$\top$}$	$\leftrightarrow$u, 14 – 21, 22 – 30
32	$a \textcircled{$\ell$} e = \textcircled{$\top$} \leftrightarrow c \textcircled{ℓ} e = \textcircled{$\top$}$	$\exists$i, 6, 7 – 31
33	$\exists y((a \textcircled{$\neg$} e = \textcircled{$\top$} \leftrightarrow y \textcircled{ℓ} e = \textcircled{$\top$}) \wedge (c \textcircled{ℓ} e = \textcircled{$\top$} \leftrightarrow y \textcircled{$\neg$} e = \textcircled{$\top$})) \rightarrow (a \textcircled{ℓ} e = \textcircled{$\top$} \leftrightarrow c \textcircled{ℓ} e = \textcircled{$\top$})$	$\rightarrow$u, 6 – 32
34	$\forall w(\exists y((a \textcircled{$\neg$} w = \textcircled{$\top$} \leftrightarrow y \textcircled{ℓ} w = \textcircled{$\top$}) \wedge (c \textcircled{ℓ} w = \textcircled{$\top$} \leftrightarrow y \textcircled{$\neg$} w = \textcircled{$\top$})) \rightarrow (a \textcircled{ℓ} w = \textcircled{$\top$} \leftrightarrow c \textcircled{ℓ} w = \textcircled{$\top$}))$	$\forall$u, 33
35	$\forall z\forall w(\exists y((a \textcircled{$\neg$} w = \textcircled{$\top$} \leftrightarrow y \textcircled{ℓ} w = \textcircled{$\top$}) \wedge (z \textcircled{ℓ} w = \textcircled{$\top$} \leftrightarrow y \textcircled{$\neg$} w = \textcircled{$\top$})) \rightarrow (a \textcircled{ℓ} w = \textcircled{$\top$} \leftrightarrow z \textcircled{ℓ} w = \textcircled{$\top$}))$	$\forall$u, 34
36	$\forall x\forall z\forall w(\exists y((x \textcircled{$\neg$} w = \textcircled{$\top$} \leftrightarrow y \textcircled{ℓ} w = \textcircled{$\top$}) \wedge (z \textcircled{ℓ} w = \textcircled{$\top$} \leftrightarrow y \textcircled{$\neg$} w = \textcircled{$\top$})) \rightarrow (x \textcircled{ℓ} w = \textcircled{$\top$} \leftrightarrow z \textcircled{ℓ} w = \textcircled{$\top$}))$	$\forall$u, 35

Treća varijanta

1	$\forall \textcircled{o}\,\forall x\forall y\exists z\; x \textcircled{o} y = z$	pretp.
2	$\forall x\forall y\; x \textcircled{ℓ} y = x$	pretp.
3	$\forall x\forall y(x \textcircled{$\neg$} y = \textcircled{$\top$} \leftrightarrow \neg x \textcircled{ℓ} y = \textcircled{$\top$})$	pretp.
4	$\forall x\forall y(x \textcircled{$\vee$} y = \textcircled{$\top$} \leftrightarrow (x \textcircled{ℓ} y = \textcircled{$\top$} \vee y \textcircled{ℓ} x = \textcircled{$\top$}))$	pretp.
5	$\forall \textcircled{$o_1$}\forall \textcircled{$o_2$}\forall \textcircled{$o_3$}\exists \textcircled{$o_4$}\forall x\forall y(x \textcircled{o_4} y = \textcircled{$\top$} \leftrightarrow \exists u\exists v((x \textcircled{o_1} y = u \wedge x \textcircled{o_2} y = v) \wedge u \textcircled{o_3} v = \textcircled{$\top$}))$	pretp.
6	$\exists y\; \textcircled{$\top$} \textcircled{$\neg$} y = c$	pretp.
7	$\forall w\; d \textcircled{ℓ} w = \textcircled{$\top$}$	pretp.
8	$\forall y(d \textcircled{$\vee$} y = \textcircled{$\top$} \leftrightarrow (d \textcircled{ℓ} y = \textcircled{$\top$} \vee y \textcircled{ℓ} d = \textcircled{$\top$}))$	$\forall$i, 4
9	$d \textcircled{$\vee$} c = \textcircled{$\top$} \leftrightarrow (d \textcircled{ℓ} c = \textcircled{$\top$} \vee c \textcircled{ℓ} d = \textcircled{$\top$})$	$\forall$i, 8
10	$d \textcircled{$\ell$} c = \textcircled{$\top$}$	$\forall$i, 7
11	$d \textcircled{$\ell$} c = \textcircled{$\top$} \vee c \textcircled{$\ell$} d = \textcircled{$\top$}$	$\vee$u, 10
12	$d \textcircled{$\vee$} c = \textcircled{$\top$}$	$\leftrightarrow$i, 9, 11
13	$\forall w\; d \textcircled{ℓ} w = \textcircled{$\top$} \rightarrow d \textcircled{$\vee$} c = \textcircled{$\top$}$	$\rightarrow$u, 7 – 12
14	$\forall y(\forall w\; y \textcircled{ℓ} w = \textcircled{$\top$} \rightarrow y \textcircled{$\vee$} c = \textcircled{$\top$})$	$\forall$u, 13
15	$\exists y\; \textcircled{$\top$} \textcircled{$\neg$} y = c \rightarrow \forall y(\forall w\; y \textcircled{ℓ} w = \textcircled{$\top$} \rightarrow y \textcircled{$\vee$} c = \textcircled{$\top$})$	$\rightarrow$u, 6 – 14
16	$\forall z(\exists y\; \textcircled{$\top$} \textcircled{$\neg$} y = z \rightarrow \forall y(\forall w\; y \textcircled{ℓ} w = \textcircled{$\top$} \rightarrow y \textcircled{$\vee$} z = \textcircled{$\top$}))$	$\forall$u, 15

Ukupno (najviše) 96 bodova.