# DRŽAVNO NATJECANJE IZ LOGIKE - RJEŠENJA

Varaždin, 23.-25. travnja 2014.

**BODOVI**:

- POTPUNO ISPRAVNO RJEŠENJE: 3 BODA
- IZOSTANAK RJEŠENJA: 1 BOD
- KRIVO ILI NEPOTPUNO RJEŠENJE: 0 BODOVA

| **ZADATAK** | **BROJ BODOVA** | **MAX BODOVA** |
|---|---|---|
| 1. | $\times$ | 30 |
| 2. | $\times$ | 30 |
| 3. | $\times$ | 24 |
| 4. | $\times$ | 21 |
| 5. | $\times$ | 42 |
| 6. | $\times$ | 69 |
| 7. | $\times$ | 57 |
| **UKUPNO** | $\times$ | **273** |

**Uputa ispravljačima:** U **2. i 4. zadatku** potrebno je obratiti pažnju na moguća alternativna rješenja koja nisu navedena u rješenjima.

**Zadatak 1.**

**Napomena:** Svaki točno ispunjen redak i svaki točno ispunjen stupac nose po tri boda.

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| A | X | X | X | CL | CK |
| B | X | X | X | CD | CP |
| C | X | X | X | X | X |
| D | X | BS | X | X | X |
| E | BL | BK | X | X | BT |

**(10×3 boda = 30 bodova)**

**Zadatak 2.**

1. $\{P, Q\}$: $P \wedge Q \rightsquigarrow P \rightsquigarrow P \vee Q$
2. $\{P, Q\}$: $P \wedge Q \rightsquigarrow \underline{/} \rightsquigarrow Q$
3. $\{P, Q, S\}$: $P \wedge Q \wedge S \rightsquigarrow \underline{P \wedge (Q \leftrightarrow S)} \rightsquigarrow \underline{P \wedge (Q \rightarrow S)} \rightsquigarrow \underline{P} \rightsquigarrow (Q \rightarrow S) \rightarrow P \rightsquigarrow \underline{P \vee Q} \rightsquigarrow \underline{P \vee Q \vee S} \rightsquigarrow \top$
4. $\{=\}$: $\exists x \forall y (x = y) \rightsquigarrow \underline{\forall x \forall y \forall z (x = y \vee y = z \vee z = x)} \rightsquigarrow \forall x \forall y \forall z \forall w (x = y \vee y = z \vee z = w \vee w = x)$
5. $\{R^2\}$: $\forall x Rxx \wedge \forall x \forall y \forall z (Rxy \rightarrow (Rxz \rightarrow Ryz)) \rightsquigarrow \underline{\forall x Rxx \wedge \forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Rxz))} \rightsquigarrow \forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Rxz))$
6. $\{P^2, S^2\}$: $\forall x \exists y \forall z (Pyx \wedge Syz) \rightsquigarrow \underline{\forall x \exists y Pyx \wedge \forall x \exists y Syx} \rightsquigarrow \underline{\exists x \exists y Pyx \wedge \forall x \exists y Syx} \rightsquigarrow \forall x \exists y Syx$

**(10×3 boda = 30 bodova)**

**Zadatak 3.**

1. $\neg(A \leftrightarrow ((A \vee B) \wedge \neg B)) \equiv \underline{A \wedge B}$

2. $\neg(A \rightarrow (\neg B \rightarrow (A \wedge B)) \equiv \underline{A \wedge \neg B}$

3. $(A \vee \neg B) \leftrightarrow ((B \vee A) \rightarrow \neg(\neg A \leftrightarrow B)) \equiv \underline{A \rightarrow B}$

4. $(B \vee (\neg B \wedge \neg A)) \rightarrow (A \wedge (\neg B \leftrightarrow A)) \equiv \underline{A \wedge \neg B}$

5. $((A \wedge \neg B) \vee (B \wedge \neg A)) \rightarrow (A \leftrightarrow ((A \wedge B) \vee \neg A)) \equiv \underline{A \leftrightarrow B}$

**Napomena:** U podzadatcima **b)**, **c)** i **d)** priznaje se samo potpuno rješenje.

**b)** (2,4), (4,2)
**c)** (1,3), (1,5), (2,4), (4,2), (5,3)
**d)** (2,3), (3,2), (3,4), (4,3)

**(8×3 boda = 24 boda)**

**Zadatak 4.**

1. $a = 0 \equiv \forall x Saxx$

2. $a = 1 \equiv \underline{\forall x Paxx}$

3. $a = 2 \equiv \underline{\exists y(\forall x Pyxx \wedge Syya)}$

4. $a = b \equiv \underline{\exists y(\forall x Syxx \wedge Sayb)}$; $\underline{\exists y(\forall x Pyxx \wedge Payb)}$

5. $a \leq b \equiv \underline{\exists x Saxb}$

6. $b$ je djeljivo s $a \equiv \underline{\exists x Paxb}$

7. $a$ je najmanji zajednički višekratnik brojeva $b$ i $c \equiv$
   $\underline{\exists x Pbxa \wedge \exists x Pcxa \wedge \forall y((\exists x Pbxy \wedge \exists x Pcxy) \rightarrow \exists x Paxy)}$;
   $\underline{\exists x Pbxa \wedge \exists x Pcxa \wedge \forall y((\exists x Pbxy \wedge \exists x Pcxy) \rightarrow \exists x Saxy)}$

8. $a$ je najveći zajednički djelitelj brojeva $b$ i $c \equiv$
   $\underline{\exists x Paxb \wedge \exists x Paxc \wedge \forall y((\exists x Pyxb \wedge \exists x Pyxc) \rightarrow \exists x Pyxa)}$;
   $\underline{\exists x Paxb \wedge \exists x Paxc \wedge \forall y((\exists x Pyxb \wedge \exists x Pyxc) \rightarrow \exists x Syxa)}$

**(7×3 boda = 21 bod)**

**Zadatak 5.**

**1.** `I, N, /, N, N, I, I`
**2.** `I, I, N, N, I, I, N`

**(14×3 boda = 42 boda)**

**Zadatak 6.**

**1. a)** Boduju se samo potpuno ispravni redci (formula ili dvije formule uz opravdanje) i svaki potpuno ispravan redak donosi tri boda. Formule u redcima 7. i 8. u lijevoj grani mogu zamijeniti mjesta.

| | | | |
|---|---|---|---|
| 1. | $\exists xFx \lor \exists xGx$✓ | | |
| 2. | $\neg\exists x(Fx \lor Gx)$✓ | | |
| 3. | $\forall x\neg(Fx \lor Gx)$✓ | | $2, \neg\exists$ |
| 4. | $\exists xFx$✓ | $\exists xGx$✓ | $1, \lor$ |
| 5. | $Fb$ | $Gb$ | $4, \exists$ |
| 6. | $\neg(Fb \lor Gb)$✓ | $\neg(Fb \lor Gb)$✓ | $3, \forall$ |
| 7. | $\neg Fb$ | $\neg Fb$ | $6, \neg\lor$ |
| 8. | $\neg Gb$ | $\neg Gb$ | $6, \neg\lor$ |
| 9. | × | × | |

**b)** Na temelju istinitosnoga stabla zaključujemo da iskaz $\exists x(Fx \lor Gx)$ `slijedi / ne slijedi` iz iskaza $\exists xFx \lor \exists xGx$.
**Napomena:** Odgovor se priznaje ako i samo ako je stablo točno riješeno.

**2. a)**

| | | | |
|---|---|---|---|
| 1. | $\exists x(Fx \lor Gx)$✓ | | |
| 2. | $\neg(\exists xFx \lor \exists xGx)$✓ | | |
| 3. | $\neg\exists xFx$✓ | | $2, \neg\lor$ |
| 4. | $\forall x\neg Fx$✓ | | $3, \neg\exists$ |
| 5. | $\neg\exists xGx$✓ | | $2, \neg\lor$ |
| 6. | $\forall x\neg Gx$✓ | | $5, \neg\exists$ |
| 7. | $Fa \lor Ga$✓ | | $1, \exists$ |
| 8. | $Fa$ | $Ga$ | $7, \lor$ |
| 9. | $\neg Fa$ | $\neg Fa$ | $4, \forall$ |
| 10. | × | $\neg Ga$ | $6, \forall$ |
| 11. | | × | |

**b)** Na temelju istinitosnoga stabla zaključujemo da iskaz $\exists xFx \vee \exists xGx$ `slijedi` / `ne slijedi` iz iskaza $\exists x(Fx \vee Gx)$. **Napomena:** Odgovor se priznaje ako i samo ako je stablo točno riješeno.
**c)** Zadani iskazi `jesu` / `nisu` ekvivalentni. **Napomena:** Odgovor se priznaje ako i samo ako su oba stabla točno riješena.

**(23×3 boda = 69 bodova)**

**Zadatak 7.**

**Napomena:** Boduju se samo potpuno ispravni redci (formula uz opravdanje) i svaki potpuno ispravan redak donosi tri boda.
**a)**

| | | |
|---|---|---|
| 1 | $\exists xPx$ | pretp. |
| 2 | $\quad Pc$ | pretp., c |
| 3 | $\quad\quad \forall x\neg Px$ | pretp. |
| 4 | $\quad\quad Pc$ | op, 2 |
| 5 | $\quad\quad \neg Pc$ | ∀i, 3 |
| 6 | $\quad \neg\forall x\neg Px$ | ¬u, 3-5 |
| 7 | $\neg\forall x\neg Px$ | ∃i, 1, 2-6 |
| 8 | $\neg\forall x\neg Px$ | pretp. |
| 9 | $\quad \neg\exists xPx$ | pretp. |
| 10 | $\quad\quad Pa$ | pretp. |
| 11 | $\quad\quad \exists xPx$ | ∃u, 10 |
| 12 | $\quad\quad \neg\exists xPx$ | op, 9 |
| 13 | $\quad \neg Pa$ | ¬u, 10-12 |
| 14 | $\quad \forall x\neg Px$ | ∀u, 13, a |
| 15 | $\quad \neg\forall x\neg Px$ | op, 8 |
| 16 | $\exists xPx$ | ¬i, 9-15 |
| 17 | $\exists xPx \leftrightarrow \neg\forall x\neg Px$ | ↔u, 1-7, 8-16 |

**b)** pouzdan
**c)** potpun

**(19×3 boda = 57 bodova)**